$$U(q,\theta,T) = v(q,\theta) - T \quad , \theta \sim F(\theta) \text{ on } [\underline{\theta}, \bar{\theta}]$$

$$\max_{\{T(q(\cdot)), q(\cdot)\}} \int_{\underline{\theta}}^{\bar{\theta}} [T(q(\theta)) - c\, q(\theta)] f(\theta) d\theta$$

$$\text{s.t.} \quad u(\theta) = v(q(\theta),\theta) - T(q(\theta)) \geq 0 \qquad (IR)$$

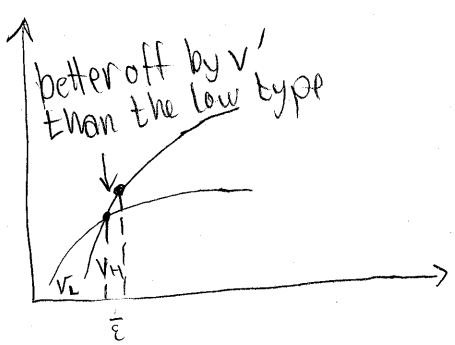
$$u(\theta) \in \arg\max_{\hat{\theta}} \{ v(q(\hat{\theta}),\theta) - T(q(\hat{\theta})) \} \qquad (IC)$$

with $\frac{\partial v(q(\theta),\theta)}{\partial \theta} > 0$, $\frac{\partial v(q(\theta),\theta)}{\partial q} > 0$

$\frac{\partial^2 v(q(\theta),\theta)}{\partial q^2} < 0$, $\frac{\partial v(q(\theta),\theta)}{\partial q \partial \theta} > 0$

By the envelope theorem, $\frac{du}{d\theta} = \frac{\partial v}{\partial \theta}$

$$\Rightarrow u(\theta) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{\partial v}{\partial x}(q(x),x)\,dx$$



T, q

$$T(q(\theta)) = \underbrace{v(q(\theta),\theta)}_{\text{gross value}} - \underbrace{u(\theta)}_{\text{net value}}$$

- monopolist extracts the rest

$$= v(q(\theta),\theta) - \underbrace{u(\underline{\theta})}_{=0} - \int_{\underline{\theta}}^{\bar{\theta}} v_\theta(q(x),x)\,dx$$

One-to-one relationship between tariff and $q(\cdot)$, so we have:

$$\max_{q(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \left[v(q(\theta),\theta) - \int_{\underline{\theta}}^{\theta} v_\theta(q(x),x)\,dx - cq(\theta) \right] f(\theta)\,d\theta$$

$$\max_{q(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \left[v(q(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q(\theta),\theta) - cq(\theta) \right] f(\theta)\,d\theta$$

FOCs: $$\frac{\partial v(q(\theta),\theta)}{\partial q(\theta)} = c + \frac{1-F(\theta)}{f(\theta)} \cdot \frac{\partial^2 v(q(\theta),\theta)}{\partial q(\theta)\,\partial\theta}$$