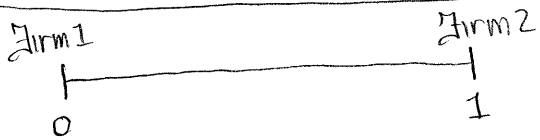


Fisher

- Static vs dynamic
  - Folk thm: anything can happen.
- Generalizing vs exemplifying theory
  - what can happen
  - io has moved away from generalizing theory.
- Collusion as central problem in io.
  - collusion takes a small role in this class
- Criticizes Nash solution concept. Why should firms play the Nash solution?
  - read some io books from the past. Nash eq. puts some discipline on the way certain topics are covered.

Bertrand vs Cournot - we won't deal with these much in this class. Cournot isn't a very good description of how most industries work. It isn't used as much now.

Product Differentiation:

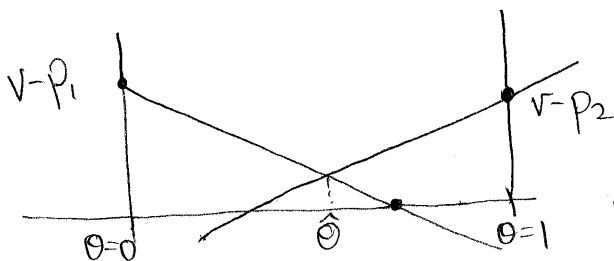


Continuum of consumers. Types  $\theta \sim U[0,1]$ .

$$u(\theta) = \begin{cases} v - p_i - d(\theta, i) & \text{if buys 1 unit from } i \\ 0 & \text{if doesn't buy} \end{cases}$$

$d(\theta, i)$  "mismatch cost":

Firm 1	Firm 2	Alternative Parametrizations
$\alpha \theta$	$\alpha(1-\theta)$	
$\alpha \theta^2$	$\alpha(1-\theta)^2$	



all  $\theta \leq \theta^1$  buy from firm 1  
 $\theta > \theta^1$  buy from firm 2.

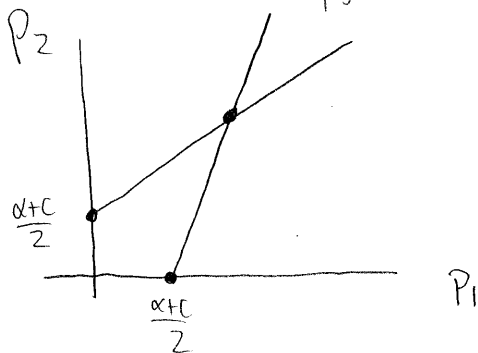
"Market is covered."

$$\hat{\theta} \text{ solves } v - p_1 - \alpha \hat{\theta} = v - p_2 - \alpha (1 - \hat{\theta})$$

$$\Rightarrow \hat{\theta} = \frac{1}{2} + \frac{p_2 - p_1}{2\alpha}$$

$$\text{Firm } i: p_i \in \operatorname{argmax}_p \left\{ (p - c) \left( \frac{1}{2} + \frac{p_j - p_i}{2\alpha} \right) \right\}$$

$$\Rightarrow BR_i(p_j) = \frac{1}{2} (\alpha + p_j + c)$$



Solving these equations simultaneously:

$$p_1^* = p_2^* = c + \alpha > c$$

$$\Rightarrow \pi_1^* = \pi_2^* = \frac{\alpha}{2}$$

as  $\alpha \rightarrow 0$ , this approaches Bertrand.

### Empirical Work

consumer  $i$  if buy good  $j$

$$u_{ij} = \begin{cases} v - \alpha p_j + \varepsilon_{ij} \\ \varepsilon_{i0} \end{cases}$$

if  $i$  consumes 1 unit of  $j$   
if  $i$  consumes nothing  
or purchase outside good.

$$\text{Type: } \varepsilon_i = (\varepsilon_{i0}, \varepsilon_{i1}, \dots, \varepsilon_{iN})$$

(Orton-Perloff (1985) examine this for  $v_1 = \dots = v_N$

and  $\varepsilon$ 's independent across  $i$  and  $j$  and no outside good.

Prop: The symmetric NE is  $p^{*(N)} = c + \frac{1}{M(N)} \cdot \frac{1}{\alpha}$

where  $M(N) = N(N-1) \int_{\mathbb{R}} F(\varepsilon)^{N-2} f(\varepsilon)^2 d\varepsilon$

where  $F$  is the cdf of  $\varepsilon_{ij}$ .

Note: The  $\alpha$  in this model is the reciprocal of the  $\alpha$  in the previous model.

Remark: Behavior is sensitive to the distribution of tastes.

Prop: If  $\varepsilon_{ij}$  has a support which is bounded above or  $\lim_{\varepsilon \rightarrow \infty} \frac{f'(\varepsilon)}{f(\varepsilon)} = -\infty$ , then  $p^{*(N)} \rightarrow c$  as  $N \rightarrow \infty$ .

◦ In uniform distribution,  $p^{*(N)} - c = O(\frac{1}{N})$

◦ This assumption holds for normal distribution.

Most common empirically:  $\varepsilon$  type I extreme value

"Logit" model  $F(\varepsilon) = e^{-e^{-\varepsilon + \eta}}$

$$\Rightarrow s_j = \frac{e^{v - \alpha p_j}}{\sum_{k=1}^N e^{v - \alpha p_k} + 1}$$

mkt share of good j

if have outside good

◦ This does not have the property that  $\lim_{\varepsilon \rightarrow \infty} \frac{f'(\varepsilon)}{f(\varepsilon)} = -\infty$ .

◦ In this model,  $p^{*(N)} = c + \frac{1}{\alpha} \frac{N}{N-1}$

◦ The results are sensitive to assumptions about distribution of types.

These are all versions of  $\frac{p-c}{p} = -\frac{1}{\varepsilon}$

$$\text{◦ In Hotelling model, } \frac{p-c}{p} = -\frac{1}{\frac{dq}{dp} \frac{p}{q}} = -\frac{1}{\frac{dq}{dp} \frac{p}{q}} = \frac{\alpha}{p}$$

$$\Rightarrow p-c = \alpha \Rightarrow p = c + \alpha$$

Everything depends on  $\frac{dQ}{dP}$ . Does it get bigger or smaller as  $N$  increases?



if this is bdd, then as  $N$  gets large, the distance b/t firms gets small  $O(\frac{1}{N})$ , so  $\frac{dQ}{dP}$  gets very large.

### Vertical Differentiation

Two firms produce qualities  $s_L$  and  $s_H$ ,  $s_H > s_L$

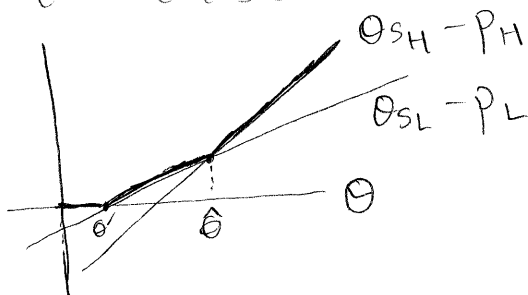
Consumers have types  $\theta \sim U[\underline{\theta}, \bar{\theta}]$

$$u(\theta) = \begin{cases} \theta s_i - p_i & \text{if buys from firm } i \\ & \text{if not buy} \end{cases}$$

Assume  $\bar{\theta} \geq 2\underline{\theta}$  and that  $c + \frac{\bar{\theta} - 2\underline{\theta}}{3}(s_H - s_L) \leq \underline{\theta}s_L$

consider the simultaneous move game:  $p_H$  and  $p_L$

are chosen



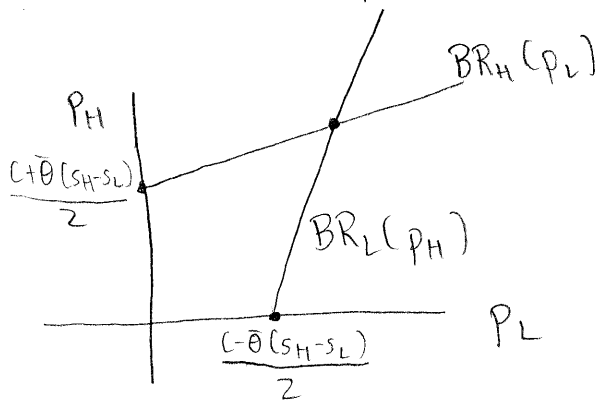
If  $\theta > \hat{\theta}$ , buy good H  
 $\theta' < \theta \leq \hat{\theta}$ , buy good L  
 $\theta \leq \theta'$ , buy nothing

Assume prices are such that  $\underline{\theta} < \hat{\theta} < \bar{\theta}$ .  
 "Market is covered"

$$\text{Then } D_H(p_L, p_H) = \bar{\theta} - \hat{\theta} = \bar{\theta} - \frac{p_H - p_L}{s_H - s_L}$$

$$\text{since } \hat{\theta} \text{ solves } \hat{\theta} s_H - p_H = \hat{\theta} s_L - p_L$$

$$\text{Also, } D_L(p_L, p_H) = \hat{\theta} - \underline{\theta} = \frac{p_H - p_L}{s_H - s_L} - \underline{\theta}$$



$$BR_H(p_L) = \operatorname{argmax}_p (p - c) \left( \bar{\theta} - \frac{p_H - p_L}{s_H - s_L} \right)$$

$$\Rightarrow BR_H(p_L) = \frac{1}{2} (p_L + c + \bar{\theta}(s_H - s_L))$$

$$BR_L(p_H) = \frac{1}{2} (p_H + c - \underline{\theta}(s_H - s_L))$$

$$p_L^* = c + \frac{\bar{\theta} - 2\underline{\theta}}{3} (s_H - s_L) \geq c \quad \text{since } \bar{\theta} \geq 2\underline{\theta}$$

$$p_H^* = c + \frac{2\bar{\theta} - \underline{\theta}}{3} (s_H - s_L) > c$$

Roughly:

1]  $p > c$  usually

2] price increases in  $\bar{\theta} - \underline{\theta}$  and in  $s_H - s_L$

Next week: Empirical paper by Bresnahan about the auto industry

## Switching Costs

Endogenous taste differences.

Two periods:  $t=1, 2$

Disutility of consuming from different firm at  $t=2$ .  
(with or without differentiation)  $\hookrightarrow$  cost of  $s$

No differentiation:

2<sup>nd</sup> period:  $p \geq c + s$  can be  $v$  or can be mixed eq.

eg  $p^* = v$

MC of serving customer at  $t=1$  is  $c - (v - c) = 2c - v$

1<sup>st</sup> period: set  $p^* < c$  to get captive customers

(\*) Paper below Bresnahan: Empirical

(\*) Judenberg and Tirole - price discrimination in second period.

Klemperer and Myerson next Wednesday