2nd degree price discrimination

Consider a model with $\theta \overset{pdf}{\sim} f(\theta)$ on $[\underline{\theta}, \bar{\theta}]$

Consumer of type θ gets utility $\theta v(q) - T$ if buys quantity/quality q at price T.

assume constant MC of c

No resale across consumers

Monopolist wants to

$$\max_{T(q),q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} [T(q(\theta)) - cq(\theta)] f(\theta) d\theta$$

$$\text{s.t.} \quad u(\theta) \geq 0 \quad \forall \theta \in [\underline{\theta}, \bar{\theta}] \quad \text{(IR)}$$

$$u(\theta) \geq \theta v(q(\theta')) - T(q(\theta')) \quad \forall \theta, \theta' \in [\underline{\theta}, \bar{\theta}] \quad \text{(IC)}$$

where $u(\theta) \equiv \theta v(q(\theta)) - T(q(\theta))$ is a value fcn.

- Guess what the binding constraints are:

  (IR'): $u(\underline{\theta}) \geq 0$

  - If lowest type gets nonnegative utility, higher types can buy the same bundle and get positive utility.

  (IC'): $u(\theta) \geq \theta v(q(\theta - d\theta)) - T(q(\theta - d\theta))$ for some $d\theta > 0$ small

  - Why does this imply $u(\theta) \geq \theta v(q(\theta - 2d\theta)) - T(q(\theta - 2d\theta))$?
    - need that $q(\theta)$ is weakly increasing
    - $\theta v'(q(\theta)) - T'(q(\theta)) \geq 0 \Rightarrow$ (IC') will hold

Reduced problem:

Note: 1] $T(q(\underline{\theta})) = \underline{\theta} v(q(\underline{\theta}))$ so that (IR') binds

2] $T'(q(\theta)) = \theta v'(q(\theta))\ \forall \theta$ so that (IC') binds

Think of monopolist as choosing $q(\theta)$. Then 1] and 2] determine $T(q(\theta))$ for any choice $q(\theta)$.

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} [T(q(\theta)) - cq(\theta)] f(\theta) d\theta$$

monopolist chooses a function

$$\text{s.t. } T(q(\underline{\theta})) = \underline{\theta}\, v(q(\underline{\theta})) \quad \forall \theta$$
$$T'(q(\theta)) = \theta v'(q(\theta)) \quad \forall \theta$$

∘ focus on gross surplus and consumer surplus.

Π = gross surplus − consumer surplus.

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Big(\underbrace{[\theta v(q(\theta)) - cq(\theta)]}_{\text{want to keep gross surplus high}} - \underbrace{u(\theta)}_{\text{and CS Low}} \Big) f(\theta) d\theta$$

$$u(\theta) = \underbrace{u(\underline{\theta})}_{=0 \text{ by (IR')}} + \int_{\underline{\theta}}^{\theta} \frac{du}{d\underset{\text{type}}{x}} dx$$

since $u(\theta) = \theta v(q(\theta)) - T(q(\theta))$

$$\frac{du}{dx} = v(q(x)) + x v'(q(x)) q'(x) - T'(q(x)) q'(x)$$
$$= v(q(x)) \quad \text{since } T'(q(\theta)) = \theta v'(q(\theta))$$

at optimal quantity (by envelope thm)

$$\underbrace{\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Big[\underbrace{\theta v(q(\theta)) - cq(\theta)}_{\substack{\text{Efficient quantity} \\ \text{would maximize this}}} - \underbrace{\int_{\underline{\theta}}^{\theta} v(q(x))dx}_{\substack{\text{want to hold x down} \\ \text{at the lower types}}} \Big] f(\theta) d\theta}_{\text{will distort down for the low types}}$$

To solve, observe:

$$\int_{\underline{\theta}}^{\bar{\theta}} \underbrace{\left(\int_{\underline{\theta}}^{\theta} v(q(x))dx\right)}_{u} \underbrace{f(\theta)d\theta}_{dv}$$

by parts
$du = v(q(\theta))$
$v = F(\theta)$

$$= \int_{\underline{\theta}}^{\bar{\theta}} (1-F(\theta)) v(q(\theta)) d\theta$$

$$\Rightarrow \max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Big\{ [\theta v(q(\theta)) - cq(\theta)] f(\theta) - (1-F(\theta)) v(q(\theta)) \Big\} d\theta$$

- there is no interaction across θs here!

$$\Rightarrow q(\theta) = \max_{q} \Big\{ [\theta v(q) - c(q)] f(\theta) - (1-F(\theta)) v(q) \Big\}$$

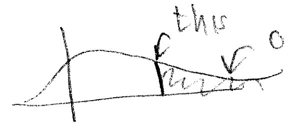
<u>FOC</u>: (q): $\theta v'(q^*(\theta)) = c + \dfrac{(1-F(\theta))}{f(\theta)} v'(q^*(\theta))$

<u>Observations:</u>

I $\theta v'(q^*(\theta)) = \underbrace{T'(q^*(\theta))}_{\text{marginal price for type } \theta}$ $\qquad p \equiv T'(q^*(\theta))$

$$\Rightarrow \frac{p-c}{p} = \frac{1-F(\theta)}{\theta f(\theta)}$$

Often, people assume "increasing hazard rate condition" $\frac{f(\theta)}{1-F(\theta)} \uparrow \theta$



over this

∘ most distributions we work with exhibit this

$\Rightarrow \frac{p-c}{p} \downarrow \theta$

2] $\frac{p-c}{p} \rightarrow 0$ as $\theta \rightarrow \bar{\theta}$ since $1-F(\theta) \rightarrow 1-1=0$ as $\theta \rightarrow \bar{\theta}$

$\Rightarrow p \rightarrow c$, "No distortion at the top."

"You don't provide excess quality to the high types, but you do underprovide quality to the low types."

3] Could have solved FOC for $v'(q^*(\theta))$

$$v'(q^*(\theta)) = \frac{c}{\theta - \frac{1-F(\theta)}{f(\theta)}}$$

4] With increasing hazard rate condition and $v''<0$

$q^*(\theta) \uparrow$ in θ ; $v'(q^*(\theta)) = \left.\frac{c}{\theta\uparrow - \underbrace{\frac{1-F(\theta)}{f(\theta)}}_{\downarrow}}\right\} \downarrow$

$\uparrow$

$\Rightarrow q^*(\theta) \uparrow$

∘ This will ensure ignored constraints are satisfied

Most papers dealing with 2^{nd} degree price discrimination use this model.

[5] average price $\frac{T(q)}{q}$ ↓ in q.

(*) price discrimination is a quantity discount system.

Damaged Goods

486 SX (486 DX had the math co-processor.)

Laser Printer E - Laser printer 10

486 SX was a damaged version of 486dx (they cut the part that went to the math co-processor.)

Laser printer E was a laser printer 10 with a delay chip.

Sample attempted model:

- High types: $2v - p$
- Low types: $v - p$

$\}$ pick v_L, v_H. But here, $c_L = c_H$.

- This is equivalent to durable goods model in which there was no price discrimination

Let $\theta \sim u[0,1]$ be the types

$\theta v_H - p$

$\theta v_L - p$

Durable goods model is $v_H = 2, v_L = 1$.

How do you get price discrimination here?
Play with the functions: $f_H(q_H)$, $f_L(q_L)$

Fudenberg, Tirole: "Behavior-based price discriminatio[n]"
- eg amazon

Shepard "Price Discrimination and Retail Configuration."

Goal: Provide evidence that firms price discriminate
- self-serve vs. full-serve gas station
  - In MA, these prices are closer together

Note:
- Distrust of cost data in IO.
- control for costs by looking at single-option stations.

Finds: $P_{SP}^{Full} - P_{SP}^{Self} = 7.6¢$

$P_{MP}^{Full} - P_{MP}^{Self} = 18.9¢$

alternatives considered:

1] Peak-load pricing
- capacity constraints in competitive markets can lead to differences in prices.

self full

full during rush hour → people spillover

2] Costs of service varying by station type
3] Econometric variation.