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Lerner index is a very important concept

Monopoly pricing

- Single product

$$\max_p \{ pD(p) - C[D(p)] \}$$

$$\Rightarrow \frac{p^m - C'}{p^m} = \frac{1}{\varepsilon}, \text{ where } \varepsilon = -\frac{\partial D(p^m)}{\partial p^m} \frac{p^m}{D(p^m)}$$

In perfect competition, the firm "sees" $\varepsilon = +\infty$

$$\Rightarrow p^m = C'$$

Multi-product:

n products

$$\max_{p_i} \left\{ \sum_{i=1}^{n} p_i D_i(p) - C(D_1(p), \dots, D_n(p)) \right\}$$

Interdependencies in: $\underbrace{D_i(p)}_{\text{demand side}}$ and $\underbrace{C}_{\text{cost side}}$

$$(p_i): \underbrace{\left(D_i + p_i \frac{\partial D_i}{\partial p_i} \right)}_{\text{MR of } i} + \underbrace{\sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i}}_{\text{MR of } -i} = \underbrace{\sum_{j=1}^{n} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i}}_{\text{MC of } i}$$

If $C(D_1(p), \dots, D_n(p)) = \sum_{i=1}^{n} C_i(D_i(p))$, FOCs become

$$(p_i) \quad \frac{p_i - c_i'}{p_i} = \frac{1}{\varepsilon_{ii}} - \sum_{j\neq i} \frac{p_j - c_j'}{p_i} \frac{D_j}{D_i} \frac{\varepsilon_{ij}}{\varepsilon_{ii}}$$

Intuition:
Higher markups if i and j are substitutes
Lower markups if i and j are complements
- Can have $p_i < c_i'$ if it helps boost sales of j!

Second degree price discrimination (Two-part tariffs)
- Self-selection of consumers (hidden types)

$\theta \in \{\theta_H, \theta_L\}$
- $T(q)$ - tariff for quality q, MC of quality is c
  - If you try to separate groups based on quantity, you typically run into arbitrage problems

- $V(q;\theta)$ value type θ derives from quality q.
- $\frac{\partial V(q;\theta)}{\partial q} > 0$; $\frac{\partial^2 V(q;\theta)}{\partial q^2} < 0$; $\frac{\partial V(q;\theta)}{\partial \theta} > 0$ $\frac{\partial^2 V(q;\theta)}{\partial \theta \partial q} > 0$

Monopolist wants to

$$\max_{q_i, T_i} \; T_L + T_H - c(q_L + q_H)$$

s.t.

$$V(q_L;\theta_L) - T_L \geq 0 \qquad (IR_L)$$
$$V(q_H;\theta_H) - T_H \geq 0 \qquad (IR_H)$$
$$V(q_L;\theta_L) - T_L \geq V(q_H;\theta_L) - T_H \qquad (IC_H)$$
$$V(q_H;\theta_H) - T_H \geq V(q_L;\theta_H) - T_L \qquad (IC_L)$$

Typically, we need to worry only about (IR_L) and (IC_H)

⇒ These should always be binding:

$$V(q_L;\theta_L) = T_L$$

$$V(q_H;\theta_H) - T_H = V(q_L;\theta_H) - \underbrace{V(q_L;\theta_L)}_{T_L}$$

$$\Rightarrow \max_{q_i} \underbrace{V(q_L;\theta_L) + V(q_H;\theta_H) - V(q_H;\theta_L)}_{T_H} + \underbrace{V(q_L;\theta_L)}_{T_L} - c(q_L+q_H)$$

$$(q_H): \frac{\partial V(q_H^*;\theta_H)}{\partial q_H} = c$$

$$(q_L): \frac{\partial V(q_L^*;\theta_L)}{\partial q_L} = c + \underbrace{\left(\frac{\partial V(q_L^*;\theta_H)}{\partial q_L} - \frac{\partial V(q_L^*;\theta_L)}{\partial q_L}\right)}_{>0 \text{ since } \frac{\partial^2 V(q;\theta)}{\partial\theta\partial q} > 0}$$

$$V(q_H^*;\theta_H) > T_H \quad \text{and} \quad V(q_L^*;\theta_L) = T_L$$

It turns out that $q_L^* < q_L^{FB}$.

Intuition:

- Lowering q_L relaxes the (IC_H) constraint. (increases the rents to the high type, so you can extract more.)

Two-part tariff

- multi-product logic
  - product 1 = "access"
  - product 2 = "consumption"

$T = A + pq$

Suppose $\theta \overset{cdf}{\sim} \overbrace{G(\theta)}^{\text{general continuous distribution}}$, $\theta \in [\underline{\theta}, \bar{\theta}]$

$V(q;\theta)$. assume $V_q > 0$, $V_{qq} < 0$, $V_{q\theta} > 0$, $V_\theta > 0$

Consumers choose:

1. consumption:

$$q(p,\theta) = \arg\max_q \{V(q;\theta) - A - pq\}$$

$$\Rightarrow V_q(q;\theta) = p$$

2. access: Consider θ^* s.t. $\theta \geq \theta^*$ enter market

$\theta^*(p,A)$ is given by $V(q(p,\theta),\theta) = pq(p,\theta) + A$

Cost side:
- access is free
- cost of c per unit

Monopolist solves

$$\max_{p,A} \Big\{ \underbrace{A[1 - G(\theta^*(p,A))]}_{\text{revenues from access}} + \underbrace{(p-c)}_{\text{markup per unit}} \underbrace{\int_{\theta^*(p,A)}^{\bar{\theta}} q(p,\theta) g(\theta) d\theta}_{\text{total quantity supplied}} \Big\}$$

$$Q = \int_{\theta^*}^{\bar{\theta}} q(p,\theta) g(\theta) d\theta = Q(p,\theta^*)$$

Let Q denote total consumption

$\frac{\partial Q}{\partial p}$ as θ^* constant

FOCs:

$$(A): 0 = [1 - G(\theta^*)] - A\, g(\theta^*)\frac{\partial \theta^*}{\partial A} + (p-c)[-q(p,\theta^*)g(\theta^*)]\frac{\partial \theta^*}{\partial A}$$

$$(p): 0 = -A g(\theta^*)\frac{\partial \theta^*}{\partial p} + Q + (p-c)\frac{\partial Q}{\partial p} + (p-c)\left[-q(p,\theta^*)g(\theta^*)\frac{\partial \theta^*}{\partial p}\right]$$

This gives

$$\frac{p-c}{p} = \frac{1}{\varepsilon_p}\left[1 - \frac{q(p,\theta^*)}{\bar{Q}}\right], \quad \bar{Q} = \frac{Q}{1-G(\theta^*)}$$

$$\frac{A - [-(p-c)q(p,\theta^*)]}{A} = \frac{1}{\varepsilon_A}$$

where $\varepsilon_A = -g(\theta^*)\frac{\partial \theta^*}{\partial A}\frac{A}{1-G(\theta^*)}$

$[-(p-c)q(p,\theta^*)]$ is interpreted as the "marginal cost of increasing A" because you lose consumers.