14.271: Industrial Organization I

Competition on a Line

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1 Horizontal Differentiation

Suppose there are two firms located at opposite endpoints of a line. Such a line can represent the actual physical location of a store or more abstractly, some space on which some consumers have preferences for one endpoint or the other. Suppose consumers have preferences given by

$$u(\theta) = \begin{cases} v - p_L - t\theta & \text{if buy from firm at left endpoint} \\ v - p_R - t(1-\theta) & \text{if buy from firm at right endpoint} \\ 0 & \text{if not buy} \end{cases},$$

where $\theta \sim U[0,1]$. Suppose prices are given by (p_L, p_R) and that the firms both have a constant marginal cost of c. Then we can find a $\hat{\theta}$ such that for all $\theta > \hat{\theta}$, a consumer of type θ purchases from the firm on the right and for all $\theta < \hat{\theta}$ purchases from the firm on the left. $\hat{\theta}$ must solve

$$\begin{aligned} v - p_L - t\hat{\theta} &= v - p_R - t\left(1-\hat{\theta}\right) \\ p_R - p_L &= 2t\hat{\theta} - t \\ 2t\hat{\theta} &= p_R - p_L + t \\ \hat{\theta} &= \frac{p_R - p_L}{2t} + \frac{1}{2}. \end{aligned}$$

Given this cutoff value, the firm on the left wants to

$$\max_{p_L} (p_L - c)\hat{\theta} = \max_{p_L} (p_L - c)\left(\frac{p_R - p_L}{2t} + \frac{1}{2}\right).$$

Taking first order conditions to derive the firm's reaction function, we have

$$(p_L): \left(\frac{p_R - p_L}{2t} + \frac{1}{2}\right) + (p_L - c)\left(-\frac{1}{2t}\right) = 0$$

or

$$\begin{aligned} p_R - p_L + t &= p_L - c \\ 2p_L &= c + t + p_R \\ p_L &= \frac{c+t}{2} + \frac{1}{2}p_R. \end{aligned} \tag{RF_L}$$

Similarly, the firm on the right wants to

$$\max_{p_R} (p_R - c)\left(1 - \hat{\theta}\right) = \max_{p_R} (p_R - c)\left(\frac{1}{2} - \frac{p_R - p_L}{2t}\right).$$

Taking first order conditions,

$$(p_R): \left(\frac{1}{2} - \frac{p_R - p_L}{2t}\right) + (p_R - c)\left(-\frac{1}{2t}\right) = 0.$$

This gives us

$$\begin{aligned} t - p_R + p_L &= p_R - c \\ 2p_R &= c + t + p_L \\ p_R &= \frac{c+t}{2} + \frac{1}{2}p_L. \end{aligned} \tag{RF_R}$$

Substituting (RF_R) into (RF_L), we have

$$\begin{aligned} p_L^* &= \frac{c+t}{2} + \frac{1}{2}\left(\frac{c+t}{2} + \frac{1}{2}p_L^*\right) \\ &= \frac{3}{4}(c+t) + \frac{1}{4}p_L^* \\ p_L^* &= c + t, \end{aligned}$$

and

$$\begin{aligned} p_R^* &= \frac{c+t}{2} + \frac{c+t}{2} \\ &= c + t. \end{aligned}$$