14.271: Industrial Organization I

Notes on Durable Goods Pricing

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1 Durable Goods with Commitment

Consider a two period model with a continuum of consumers of type $\theta \sim U[0,1]$. A consumer of type θ gets utility

$$u(\theta) = \begin{cases} 2\theta - p_1 & \text{if buy at } t = 1 \\ \theta - p_2 & \text{if buy at } t = 2 \\ 0 & \text{if not buy} \end{cases}.$$

Suppose marginal cost is constant at c. In order to solve this model, we determine the cut-off value $\hat{\theta}_1$ for which consumers are indifferent between buying at $t = 1$ and buying at $t = 2$ and the cut-off value $\hat{\theta}_2$, for which consumers are indifferent between buying at $t = 2$ and not buying. That is, $\hat{\theta}_1$ solves

$$\begin{aligned} 2\hat{\theta}_1 - p_1 &= \hat{\theta}_1 - p_2 \\ \hat{\theta}_1 &= p_1 - p_2 \end{aligned}$$

and

$$\begin{aligned} \hat{\theta}_2 - p_2 &= 0 \\ \hat{\theta}_2 &= p_2. \end{aligned}$$

Thus, $p_1 = \hat{\theta}_1 + p_2 = \hat{\theta}_1 + \hat{\theta}_2$. The firm then chooses $\hat{\theta}_1$ and $\hat{\theta}_2$ to

$$\begin{aligned} & \max_{\substack{\hat{\theta}_1, \hat{\theta}_2 \\ \hat{\theta}_1 \geq \hat{\theta}_2}} (p_2 - c)\left(\hat{\theta}_1 - \hat{\theta}_2\right) + (p_1 - c)\left(1 - \hat{\theta}_1\right) \\ = & \max_{\substack{\hat{\theta}_1, \hat{\theta}_2 \\ \hat{\theta}_1 \geq \hat{\theta}_2}} \left(\hat{\theta}_2 - c\right)\left(\hat{\theta}_1 - \hat{\theta}_2\right) + \left(\hat{\theta}_1 + \hat{\theta}_2 - c\right)\left(1 - \hat{\theta}_1\right). \end{aligned}$$

Taking first order conditions, ignoring the $\hat{\theta}_1 \geq \hat{\theta}_2$ constraint for now,

$$\begin{aligned} \left(\hat{\theta}_1\right) &: \left(\hat{\theta}_2 - c\right) + \left(1 - \hat{\theta}_1\right) - \left(\hat{\theta}_1 + \hat{\theta}_2 - c\right) = 0 \\ \left(\hat{\theta}_2\right) &: \left(\hat{\theta}_1 - \hat{\theta}_2\right) - \left(\hat{\theta}_2 - c\right) + \left(1 - \hat{\theta}_1\right) = 0. \end{aligned}$$

$\left(\hat{\theta}_1\right)$ gives us

$$\begin{aligned} \hat{\theta}_2 - c + 1 - \hat{\theta}_1 &= \hat{\theta}_1 + \hat{\theta}_2 - c \\ \hat{\theta}_1 &= \frac{1}{2}. \end{aligned}$$

Plugging this into $\left(\hat{\theta}_2\right)$, we have

$$\begin{aligned} \left(\frac{1}{2} - \hat{\theta}_2\right) - \left(\hat{\theta}_2 - c\right) + \left(1 - \frac{1}{2}\right) &= 0 \\ -2\hat{\theta}_2 + c + 1 &= 0 \\ \hat{\theta}_2 &= \frac{1}{2} + \frac{c}{2}. \end{aligned}$$

Thus, we see that $\hat{\theta}_2 > \hat{\theta}_1$, which is a contradiction. Thus, the $\hat{\theta}_1 \geq \hat{\theta}_2$ constraint must be binding, and we have that $\hat{\theta}_1 = \hat{\theta}_2$. (i.e. we have no one who wants to buy at period 2 but not at period 1.) This gives us that $p_2 = \frac{1}{2}$ and $p_1 = \frac{1}{2} + \frac{1}{2} = 1$.

2 Durable Goods without Commitment

As above, consider a two period model with a continuum of consumers of type $\theta \sim U[0,1]$. A consumer of type θ gets utility

$$u(\theta) = \begin{cases} 2\theta - p_1 & \text{if buy at } t = 1 \\ \theta - p_2 & \text{if buy at } t = 2 \\ 0 & \text{if not buy} \end{cases}.$$

Here, I will assume that marginal cost is constant and $c = 0$. In the second period, the firm, having sold to all consumers with $\theta \geq \hat{\theta}_1$ in the first period, would like to decrease its second period price to capture more of the market in the second period. The firm would like to commit to not lowering its prices in the second period, but this is not a credible commitment, in the subgame perfect Nash equilibrium sense. In the previous section, we assumed that the firm, indeed, could make such a commitment. Here, we will derive some results for the situation in which the firm cannot make such a commitment.

As with any sequential move game of complete information (on the part of the firm), we proceed by backward induction. Suppose at $t = 1$, the firm sells to all $\theta \geq \theta_1$. Then at $t = 2$, the firm faces the demand curve given by $\theta_1 - p_2$. The firm's problem is then to

$$\pi_2(\theta_1) = \max_{p_2} p_2 (\theta_1 - p_2),$$

which gives us

$$\begin{aligned} (p_2) \quad &: \quad \theta_1 - p_2^* - p_2^* = 0 \\ p_2^* \quad &= \quad \frac{\theta_1}{2}. \end{aligned}$$

The second period profits are thus

$$\pi_2(\theta_1) = \frac{\theta_1}{2}\frac{\theta_1}{2} = \frac{\theta_1^2}{4}.$$

Next, at $t = 1$, we have that θ_1 will be such that consumers of type θ_1 are indifferent between purchasing at $t = 1$ and at $t = 2$. That is, θ_1 will solve

$$2\theta_1 - p_1 = \theta_1 - p_2^* = \frac{\theta_1}{2},$$

so $\theta_1 = \frac{2}{3}p_1$. (i.e. all consumers with $\theta \geq \theta_1$ will purchase in the first period) The first period firm then wants to

$$\begin{aligned} \pi^{nc} &= \max_{p_1} p_1 (1 - \theta_1) + \pi_2(\theta_1) = \max_{p_1} p_1 \left(1 - \frac{2}{3}p_1\right) + \frac{\left(\frac{2}{3}p_1\right)^2}{4} \\ &= \max_{p_1} p_1 \left(1 - \frac{2}{3}p_1\right) + \frac{p_1^2}{9}. \end{aligned}$$

The first order conditions are

$$(p_1) : 1 - \frac{2}{3}p_1^* - \frac{2}{3}p_1^* + \frac{2}{9}p_1^* = 0$$

$$\begin{aligned} 1 + \frac{2-12}{9}p_1^* &= 0 \\ p_1^* &= \frac{9}{10}. \end{aligned}$$

Which then gives us

$$\theta_1^* = \frac{2}{3}p_1^* = \frac{3}{5}$$

and

$$p_2^* = \frac{1}{2}\theta_1^* = \frac{3}{10}.$$

Thus, we have a decreasing price sequence $(p_1^*, p_2^*) = \left(\frac{9}{10}, \frac{3}{10}\right)$, and we sell to consumers in both periods. What are the profits of the firm in this situation?

$$\begin{aligned}
\pi^{nc} &= p_1^* (1 - \theta_1^*) + \frac{\theta_1^*}{4} \\
&= \frac{9}{10}\left(1 - \frac{3}{5}\right) + \frac{\left(\frac{3}{5}\right)^2}{4} = \frac{9}{20}.
\end{aligned}$$

How does this compare to the maximized profits in the previous section in which the firm is able to commit to not cutting prices in the second period?

$$\begin{aligned}
\pi^c &= p_1^c \left(1 - \hat{\theta}_1^c\right) + p_2^c \underbrace{\left(\hat{\theta}_2^c - \hat{\theta}_1^c\right)}_{=0} \\
&= 1\left(1 - \frac{1}{2}\right) = \frac{1}{2}.
\end{aligned}$$

Thus, $\pi^{nc} = 0.45 < 0.5 = \pi^c$, and we have that this inability to commit in the second period is harmful for the firm. Another way of thinking about this is in terms of unappropriated externalities. Suppose the firm owner sells the firm at the end of the first period. The purchaser is, of course, willing to pay up to π_2 for the firm. (We are assuming no discounting.) In the second period, the new firm owner maximizes his profits. However, he does not take into account the externalities he is imposing on the first period firm owner. That is, he does not account for the fact that by undercutting the first period price, he is now attracting customers who otherwise would have bought in the first period. This failure to take into account these negative externalities will result in increased output in the second period. Had the first firm owner not sold his firm, he would have been able to internalize these externalities and produce the efficient (from his perspective) quantity in both periods, leading to a higher stream of profits.