Send Ufuk an e-mail for dynamic optimization notes.

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t. } k_{t+1} = f(k_t) - c_t \quad \forall t$$

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \mu_t (f(k_t) - c_t - k_{t+1})$$

$$(c_t): \quad \beta^t u'(c_t) - \mu_t = 0 \Rightarrow \mu_t = \beta^t u'(c_t)$$

$$(k_{t+1}): \quad -\mu_t + \mu_{t+1} f'(k_{t+1}) = 0 \Rightarrow \mu_t = \mu_{t+1} f'(k_{t+1})$$

Combining, we have $\beta^t u'(c_t) = \beta^{t+1} u'(c_{t+1}) f'(k_{t+1})$

$$\Rightarrow \frac{u'(c_t)}{u'(c_{t+1})} = \beta f'(k_{t+1}) \quad \text{Euler Equation}$$

u is concave $\Rightarrow$ u' is decreasing

$$\beta \uparrow \Rightarrow \frac{u'(c_t)}{u'(c_{t+1})} \uparrow \Rightarrow u'(c_t) \uparrow \text{ and } u'(c_{t+1}) \downarrow$$

$$\Rightarrow c_t \downarrow, \; c_{t+1} \uparrow$$

Dynamic Programming approach:

$$V(k_t) = \max_{c_t} \{u(c_t) + \beta V(k_{t+1}) : k_{t+1} = f(k_t) - c_t\}$$

$$= \max_{c_t} \{u(c_t) + \beta V(f(k_t) - c_t)\}$$

FOC: (c_t): $u'(c_t) - \beta V'(\overbrace{f(k_t) - c_t}^{k_{t+1}}) = 0$

EC: $V'(k_t) = u'(c_t) f'(k_t)$

Thus, $u'(c_t) = \beta V'(k_{t+1})$

$V'(k_{t+1}) = u'(c_{t+1}) f'(k_{t+1})$

$$\Rightarrow \frac{u'(c_t)}{u'(c_{t+1})} = \beta f'(k_{t+1})$$

Uncertainty and Choice:

Let $s \in \{1, \dots, m\}$ be the state of the world.

Define $p_i = \Pr\{s = i\}$

$p_i \geq 0, \quad \sum_{i=1}^{m} p_i = 1$

Let Y_i be the outcome in state i

Objective function: $F(Y_1, \dots, Y_m; p_1, \dots, p_m)$

Under VNM assumptions, $F(Y_1, \dots, Y_m; p_1, \dots, p_m) = \sum_{i=1}^{m} p_i U(Y_i)$

But there are problems with this.

Risk aversion: $m = 2$

$U(pY_1 + (1-p)Y_2) > pU(Y_1) + (1-p)U(Y_2)$

$U(E[Y]) > E[U(Y)]$

ie U is concave: $U'' < 0$ if $U \in C^2$.

$L(x) = (Y_1 - x + bx, Y_2 - x; p, 1-p)$

$\max_x \quad pu(Y_1 - x + bx) + (1-p)u(Y_2 - x)$

(x): $(1-b)pu'(Y_1 - x + bx) = (1-p)u'(Y_2 - x)$