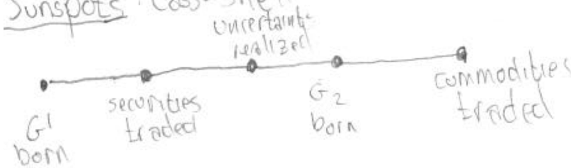
Sunspots: Cass-Shell



Incomplete participation in markets if $G_2 \neq \emptyset$.

Extrinsic uncertainty is uncertainty that does not affect the economic fundamentals (eg. endowments, preferences)

a sunspot equilibrium is an equilibrium in which allocations depend on the outcome of extrinsic uncertainty.

Arrow Sequential Markets
- meeting every period

$\Longleftrightarrow$

Debreu Contingent Claims
- meet at the beginning of time

if 1) $\exists$ complete set of arrow securities.
2) VNM utility

Thm 1: If $G_2 = \emptyset$, then sunspots do not matter

Thm 2: Sunspot equilibria are Pareto inefficient

Thm 3: If $G_2 \neq \emptyset$, there may exist equilibria in which sunspots matter.

Indeterminacy $\not\Leftrightarrow$ sunspots

Why does 1st welfare theorem break down in Samuelson economy?

Shell: double infinity of agents and goods.

Example

$\max E_t \left[c_{t+1} - \frac{n_t^2}{2} \right]$ optimization problem for G^t

st $p_t n_t \geq M_t$, $p_{t+1} c_{t+1} \leq M_t$

Production technology: $y_t = n_t$

By nonsatiation, the constraints will be binding:

$$\left.\begin{array}{l} M_t = p_t n_t \left[= p_t y_t \Rightarrow \frac{M_t}{p_t} = y_t \right] \\ M_t = p_{t+1} c_{t+1} \end{array}\right\} p_{t+1} c_{t+1} - p_t n_t = 0 \Rightarrow c_{t+1} = \frac{p_t}{p_{t+1}} n_t$$

$$G^0: \quad \frac{M_0}{p_1} = c_1$$

Substituting BC into objective fn:

$$\max_{n_t} E_t \left[\frac{p_t}{p_{t+1}} n_t - \frac{n_t^2}{2} \right]$$

FOC:

$$(n_t) \quad E_t \left[\frac{p_t}{p_{t+1}} - n_t \right] = 0$$

note: n_t, p_t are known at time t

$$\Rightarrow n_t^* = E_t \left[\frac{p_t}{p_{t+1}} \right]$$

Suppose $M_t = M_{t-1} + \underbrace{g p_t}_{\text{amount of money the government prints to finance its consumption}}$

Economy-wide resource constraint at time t:

$$y_t = c_t + g \overset{y_t = n_t}{\Longrightarrow} n_t = c_t + g$$

RC: $n_t = c_t + g$

Gov't BC: $M_t = M_{t-1} + g p_t$

$$\Rightarrow \frac{M_t}{p_t} = \frac{M_{t-1}}{p_{t-1}} \frac{p_{t-1}}{p_t} + g \Rightarrow \frac{\frac{M_t}{p_t} - g}{M_{t-1}/p_{t-1}} = \frac{p_{t-1}}{p_t} \qquad (1)$$

$$n_t = E_t \left[\frac{p_t}{p_{t+1}} \right] \qquad (2)$$

Recall: $c_{t+1} = \frac{p_t n_t}{p_{t+1}}$

$p_{t+1} c_{t+1} = M_t \Rightarrow c_{t+1} = \frac{M_t}{p_{t+1}}$

$$\frac{M_t}{p_{t+1}} = \frac{p_t}{p_{t+1}} n_t$$

$$\Rightarrow \frac{M_t}{p_t} = n_t \quad (3)$$

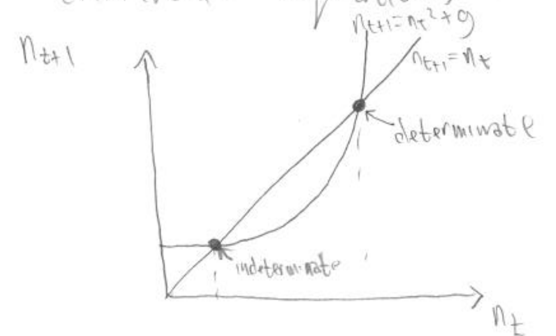
Substituting (1) into (2):

$$n_t = E_t \left[\frac{M_{t+1}/p_{t+1} - g}{M_t / p_t} \right]$$

Substituting (3) into this:

$$n_t = E_t \left[\frac{n_{t+1} - g}{n_t} \right] \Rightarrow n_t^2 = E_t [n_{t+1}] - g \quad (4)$$

Looking at (4) without the expectation (nonstochastic difference equation): $n_t^2 = n_{t+1} - g \Rightarrow n_{t+1} = n_t^2 + g$



assuming $n_t \in [0, \overline{B}]$ (ie finite upper bound), we cannot have explosive equilibrium

(*) Note: Labor hours are on the axes instead of $\frac{1}{\pi_{t+1}}$ as in previous OLG models

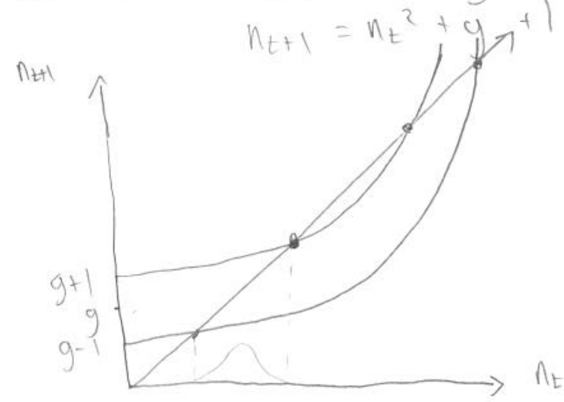
Now, what happens if we introduce sunspot variable?

$n_t^2 = n_{t+1} + \delta_{t+1} - g$ where $\delta_{t+1} = \begin{cases} 1 & \text{if sunspot} \\ -1 & \text{else} \end{cases}$

$E[\delta_{t+1}] = 0$

In one case: $n_{t+1} = n_t^2 + g - 1$ if $\delta_{t+1} = 1$

$n_{t+1} = n_t^2 + g + 1$ if $\delta_{t+1} = -1$



$n_t^2 = E_t[n_{t+1} + \delta_{t+1}] - g$ $\quad$ δ_{t+1} is a mean zero sunspot shock

$= E_t[n_{t+1}] - g + E_t[\delta_{t+1}]$

$= E_t[n_{t+1}] - g$

$\Rightarrow$ This is consistent with rational expectations

Classical vs. Samuelson (Gale)

Kehoe-Levine

Cass-Shell

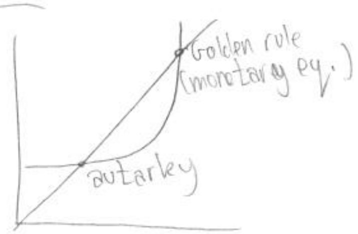
Debreu-Mantel-Sonnenschein

High probability:

- QZ decomposition - gen'l eits, on matrices,
- QΔ
- Solving Rational expectations models

  Generalized eigenvalues solve $Ax = \lambda Bx$.

- Markov stuff



Kehoe-Levine: Finite odd number of each of these type of equilibria

└ Indeterminacy in a class of these models.