$$GPA_i = \alpha + \beta\, income_i + \varepsilon_i \qquad i=1,\dots,8$$

$$\left.\begin{array}{l}\sum_{i=1}^{8}(X_i-\bar{X})(Y_i-\bar{Y}) = 1950 \\ \sum_{i=1}^{8}(X_i-\bar{X})^2 = 1950\end{array}\right\} \beta = 1$$

$$(2)\quad \sum_{i=1}^{8}\left[(Y_i-\bar{Y}) - \frac{\sum_{i=1}^{8}(X_i-\bar{X})(Y_i-\bar{Y})}{\sum_{i=1}^{8}(X_i-\bar{X})}(X_j-\bar{x})\right]^2 = 0.6528$$

$$\hat{\beta} = \frac{\sum_{i=1}^{8}(X_i-\bar{X})(Y_i-\bar{Y})}{\sum_{i=1}^{8}(X_i-\bar{X})^2} = \frac{19.50}{19.50} = 0$$

$$\begin{aligned}(2) &= \sum_{i=1}^{8}\left[Y_i-\bar{Y}-\hat{\beta}(X_i-\bar{X})\right]^2 \\ &= \sum_{i=1}^{8}\left[Y_i-\bar{Y}-\hat{\beta}X_i+\hat{\beta}\bar{X}\right]^2 \\ &= \sum_{i=1}^{8}\left[Y_i-\hat{\beta}X_i-\underbrace{(\bar{Y}-\hat{\beta}\bar{X})}_{\hat{\alpha}}\right]^2 \\ &= \sum_{i=1}^{8}(Y_i-\hat{\alpha}-\hat{\beta}X_i)^2 = \sum_{i=1}^{8} e_i^2\end{aligned}$$

$$\widehat{se(\hat{\beta})} = \sqrt{s^2(\Sigma x_i^2)^{-1}} = \sqrt{\frac{1}{6}0.6528 \cdot \frac{1}{19.50}} = \sqrt{\frac{.1088}{19.50}} \approx .074$$

$$|t| = \left|\frac{1}{0.074}\right| \approx |13| > 2.7 = t_c(8-2) = t_c(6) \Rightarrow \text{reject } H_0: \beta = 0$$

$$t = \left|\frac{\hat{\beta}_2+\hat{\beta}_3-1}{se(\hat{\beta}_2,\hat{\beta}_3)}\right| = \left|\frac{0.18+1.05-1}{\sqrt{.0044+.0585-0.0294}}\right| = \left|\frac{0.23}{\sqrt{.0335}}\right| \approx 1.28$$

$\ln Y_i = \beta_1 + \beta_2 \ln K_i + \beta_3 \ln L_i + \varepsilon_i$

$H_0: \beta_2 + \beta_3 = 1 \iff \beta_2 = 1 - \beta_3$

Restricted:

$\ln Y_i = \beta_1 + \ln K_i + \beta_3 (\ln L_i - \ln K_i) + \varepsilon_i$

$\ln Y_i - \ln K_i = \beta_1 + \beta_3 (\ln L_i - \ln K_i) + \varepsilon_i$

F statistic: $\frac{[ESS_R - ESS_{UR}]/q}{(ESS_{UR})/(N-k)} \approx F$ of $F(q, n-k)$

$$F = \frac{(.017 - .016)/1}{(.016)/16} = \frac{.001}{.001} = 1$$